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ABSTRACT

A model of test item dependency is presented and used to illustrate the effect that violations of local independence have on the behavior of item characteristic curves. The dependency model is flexible enough to simulate the interaction of a number of factors including item difficulty and item discrimination, varying degrees of item dependence, and item order or sequence effects. The model also provides for an ability-by-dependence interaction. Results suggest that the shift in an item's characteristic curve can be fairly dramatic, producing nonlogistic response probability curves. (Author/GDC)

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# A General Model for Item Dependency

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Running Head: ITEM DEPENDENCY MODEL

TM 860 496

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Abstract

The purpose of this paper is to define a model of item dependency and to use it to illustrate the effect that violations of local independence have on the behavior of item characteristic curves. The dependency model is flexible enough to simulate the interaction of a number of factors including item difficulty and discrimination, varying degrees of item dependence and item order or sequence effects. The model also provides for an ability-by-dependence interaction. Results suggest that the shift in an item's characteristic curve can be fairly dramatic, producing nonlogistic response probability curves. Suggestions for future work with the model and its implications are made.

## A General Model for Item Dependency

Local independence is an assumption of all of the current item response theory (IRT) models. The violation of this assumption certainly can occur in situations where items are nested (i.e., multiple part questions where the correct solution to one part or item must be achieved before the solution to subsequent parts can be achieved -- see the example in Figure 1). Furthermore, the violation of the independence assumption could occur in other situations which are much less obvious. In fact one of the justifications for considering IRT models that would allow for the violation of the local independence assumption would be to be able to detect and possibly eliminate or modify items that would inadvertently "cue" the correct response to other items on the same test. The purpose of this paper is to define a model of item dependency (in which local independence between item responses is a special case) and use it to illustrate what happens to typical test items, as represented by the behavior of their item characteristic curves (ICC's) when varying degrees of dependency exist.

The item dependency model proposed is flexible enough to simulate or describe the interaction of a number of factors such as item difficulty and discrimination, the amount and direction of the dependency, and item order or sequence effects. An underlying assumption of the model is that each item within a sequence of k items is dependent only on the previous item (i.e., a "one-item-back" dependency, the simplest situation). However, other "item-lags" are possible to describe with the proposed model.

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Insert Figure 1 About Here

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## Definition of the Model

The item dependency model is defined as follows. Let $P_j(\theta_i)$ represent the probability of an examinee with trait measure, $\theta_i$, answering test item j correctly, independently of any other test item. Later, we will assume that $P_j(\theta_i)$ is determined by a particular function of $\theta$, but for the present discussion, simply define $P_j(\theta_i)$ as a probability measure. To link the items in a dependent fashion, define transition probabilities between two items, j and j-1, as pictured below.

| | | j = 0 | j = 1 |
|---|---|---|---|
| j-1 | 0 | $1 - \alpha^*_{ij}$ | $\alpha^*_{ij}$ |
| | 1 | $\beta^*_{ij}$ | $1 - \beta^*_{ij}$ |

In this model, $\alpha^*_{ij}$ represents the probability that an examinee with trait $\theta_i$ will move from an incorrect response on item j-1 (state 0) to a correct response on item j (state 1). Similarly, $\beta^*_{ij}$ represents a transition probability from a correct response on item j-1 to an incorrect response on item j. The probabilities, $1 - \alpha^*_{ij}$ and $1 - \beta^*_{ij}$ imply state consistency between items.

We note that items j and j-1 are assumed to be adjacent test items only for the purpose of discussion in this paper. This is not a requirement, however, and in fact all discussion may be generalized to any two test items, j and j-$\tau$, where $\tau = 1,2,\ldots,k-1$ and $j = \tau+1,\tau+2,\ldots,k$.

These four cell probabilities are functions of (1) the jth item-by-ith trait interaction, as given by $P_j(\theta_i)$, and (2) the amount and direction of any item dependency. This definition of the transition probabilities is similar in structure to the latent Markov chain model described by Lazarsfeld and Henry (1968). These probabilities are defined as follows.

$$\alpha^*_{ij} = \alpha P_j(\theta_i)$$

and

$$\beta^*_{ij} = \beta Q_j(\theta_i)$$

where

$$Q_j(\theta_i) = 1 - P_j(\theta_i).$$

The parameters, $\alpha$ and $\beta$, are used as dependency weights, with $0 \leq \alpha \leq 1$ and $0 \leq \beta \leq 1$. For the purpose of the simple examples provided in this paper, the weights are assumed to be equal for all adjacent pairs of items in the k-item sequence. This is not required, however; within one k-item sequence, $\alpha$ and $\beta$ may take on any of the values in the range described above between any two pairs of items, and $\alpha$ and $\beta$ need not be equal to one another.

Once the transition probabilities have been defined, the complement probabilities can be written as

$$1 - \alpha^{*}_{ij} = 1 - \alpha P_j(\theta_i)$$

and

$$1 - \beta^{*}_{ij} = 1 - \beta Q_j(\theta_i).$$

The dependency weights, $\alpha$ and $\beta$, fix the amount of dependency among the k items, and since they can be assigned values independently of one another, they also fix the direction of item dependency (e.g., from correct to incorrect). When $\alpha = \beta = 1$, the items are independent, and when $\alpha = \beta = 0$, the items are completely dependent. This is more easily seen from the definition of the success probability for item j that results from the item dependency of previous items, or $P'_j(\theta_i)$. $P'_j(\theta_i)$ is the probability of answering the jth item correctly, given an incorrect response to the previous item or given a correct response to the previous item. In other words,

$$P'_j(\theta_i) = Q'_{j-1}(\theta_i)\alpha^{*}_{ij} + P'_{j-1}(\theta_i)\{1 - \beta^{*}_{ij}\}$$

$$= Q'_{j-1}(\theta_i)\alpha P_j(\theta_i) + P'_{j-1}(\theta_i)\{1 - \beta\, Q_j(\theta_i)\} \quad . \qquad (1)$$

When $\alpha = \beta = 1$,

$$P'_j(\theta_i) = Q'_{j-1}(\theta_i)P_j(\theta_i) + P'_{j-1}(\theta_i)\{1 - Q_j(\theta_i)\}$$

$$= \{1 - P'_{j-1}(\theta_i)\}P_j(\theta_i) + P'_{j-1}(\theta_i)P_j(\theta_i)$$

$$= P_j(\theta_i) - P_j(\theta_i)P'_{j-1}(\theta_i) + P'_{j-1}(\theta_i)P_j(\theta_i)$$

$$= P_j(\theta_i) \ .$$

This implies that the response for the jth item, given $\theta_i$, depends only on the jth item's ICC.

Similarly, when $\alpha = \beta = 0$, $P_j(\theta_i) = P'_{j-1}(\theta_i)$ and the jth item response is solely determined by the previous item probability of a correct response. That is, no characteristics of the jth item have any influence on the correct response for item j.

This model is a Markov chain with nonstationary transition probabilities (i.e., the transition probabilities remain constant only between pairs of items for an examinee with trait measure, $\theta_i$). However, the joint probability distribution for k dependent items within the same sequence can still be written as with a Markov chain with stationary transition probabilities. Specifically, if $U_j$ is the response variable for the jth item (with the examinee or trait index deleted for notation simplicity), where

$$u_j = \begin{cases} 1, \text{ for a correct response} \\ 0, \text{ otherwise,} \end{cases}$$

then the joint probability distribution for a given examinee for items 1 and 2 within the sequence of k dependent items can be written as

$$P(U_1,U_2) = P(U_2|U_1)\ P(U_1) \ .$$

For items 1, 2, and 3, the joint probability distribution can be written as

$$P(U_1,U_2,U_3) = P(U_3|U_1,U_2)\ P(U_1,U_2)$$

$$= P(U_3|U_1,U_2)\ P(U_2|U_1)\ P(U_1)$$

$$= P(U_3|U_2)\ P(U_2|U_1)\ P(U_1)$$

and for k items,

$$P(U_1,U_2,\ldots,U_k) = P(U_k|U_{k-1})\ P(U_{k-1}|U_{k-2})\ldots P(U_1). \qquad (2)$$

In this manner, the joint probability distribution for k items can be "built up" as products of the probability distribution of the first item and the appropriate, subsequent transition probabilities of the remaining item pairs. This is similar to the construction of the joint probability distribution for a Markov chain with stationary transition probabilities (Ross, 1976).

From this definition of a joint probability distribution, it is easy to see that equation (1) is actually the marginal probability of answering item j correctly, or is the ICC for item j, given that there is some dependence on item j-1, since

$$P(U_j=1) = \sum_{i=0}^{1} P(U_j=1,U_{j-1}=i)$$

$$= \sum_{i=0}^{1} P(U_j=1|U_{j-1}=i)\ P(U_{j-1}=i)\ .$$

Specifically, in the case of the proposed model,

$$P(U_j=1) = P(U_{j-1}=0)\alpha_j^* + P(U_{j-1}=1)\{1-\beta_j^*\}$$

which is equivalent to equation (1).

Therefore, we can study the influence of the dependency between item responses on an item's ICC by graphing equation (1) as the dependency weights, $\alpha$ and $\beta$, vary. In addition if some specific structure defines $P_j(\theta_i)$ (e.g., the 1-,2- or 3-parameter logistic function) then we can also study the effects that item difficulty, discrimination and pseudo-chance interactions with the dependency have on the resulting ICC, $P_j'(\theta_i)$.

Dependency Influences on ICC's

For the sake of comparison, we have included a graphical interpretation of the influence of item dependency on the ICC's of 1-,2- and 3-parameter logistic models, abbreviated as 1-,2- and 3-PLM. However, discussion primarily will center around the 2-parameter model. Table 1 gives the parameter values for three items in terms of their independent ICC's (i.e., in terms of what the parameters would be if the items were locally independent of each other).

.....

Insert Table 1 Here

.....

The ICC's for all three items in a hypothetical three-item sequence are of the general form

$$P_j(\theta_i) = c_j + (1-c_j)/\{1+\exp[+1.7a_j(b_j-\theta_i)]\}$$

where $a_j$ and $b_j$ are the item discrimination and difficulty parameters for the jth item and $c_j$ is the pseudo-chance parameter which is zero in the 1- and 2-parameter models. Many of these results are best presented graphically. Figure 2 shows how three test items in this sequence, consisting of an easy, a medium and a difficult item, would behave as $\alpha$ and $\beta$, in increments of .1, go from the independent condition to the completely dependent case. In each situation we have let $\alpha = \beta$. However, as mentioned previously, this is not required.

.....

Insert Figure 2 Here

.....

The first item in the sequence (ITEM #1) was unaffected by any other item and thus its ICC was determined solely by its initial item parameters, $a_1$ and $b_1$. The second item's family of ICC's is shown at the top right of Figure 2. When $\alpha = \beta = 1$, the item was totally independent of ITEM #1 and the first item had no effect. When $\alpha = \beta = 0$, the item was totally dependent on the previous item and the ICC of ITEM #2, in fact, was that for ITEM #1. This implied that ITEM #2's difficulty and discrimination parameters had no bearing on how an examinee responded to that item, but rather the probability of a correct response was the same as in ITEM #1.

The general result of these cases of dependency for ITEM #2 was to make the item easier than it would have been had local independence held.

Furthermore, the item was generally less discriminating than it would have been in the independent case.

The third item was the most difficult of the three, and the independent ICC for this item is shown as the lowest curve in the bottom half of the figure. Once again, as the degree of dependency strengthened (i.e., as $\alpha = \beta$ approached 0), the ICC became increasingly distorted until, in the totally dependent case, it was identical to the ICC for ITEM #1 (i.e., ITEM #3 looked like ITEM #2 which looked like ITEM #1). Of course, keep in mind that we have set $\alpha$ and $\beta$ values to be the same on each item pair in the sequence. Thus, when $\alpha = \beta = 0$ between ITEM #1 and #2, and $\alpha = \beta = 0$ between ITEM #2 and ITEM #3, the ICC for ITEM #3 became that of ITEM #1 rather than ITEM #2. Again, this is not a requirement of the model and in those cases where the values of $\alpha$ and $\beta$ are not the same across the different pairs of items within the same sequence, this ICC identity for all items would not hold.

The effect of this dependency on the third item within this sequence was to make the item easier for all examinees and to make the item, overall, less discriminatory. These tendencies were also evident in the 1- and 3-parameter models (See Figures 3 and 4). Note the ICC distortion, especially for strong cases of dependency. The resulting ICC's, although still monotonic, became nonlogistic in shape.

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Insert Figures 3 and 4 Here

.....

Item Order or Sequence Effects of the ICC

It should be noted that the order of the items is also important. If an easy item were to follow a more difficult one with some dependency between the items, the resulting ICC of the dependent item would be more difficult than the independent case. This order effect is shown in Figures 5, 6 and 7 in conjunction with Table 2. The table lists six different item sequences based on the possible permutations of the three items (Easy, b = -2.3; Medium, b = 0.0; Difficult, b = 2.0). Each figure gives the four different ICC's (based on the 2-parameter model) for the three items, depending upon each item's position within the sequence, either second or third. The solid ICC on each graph represents the item's independent ICC while the family of three dotted lines represents situations where $\alpha = \beta = .4$, .5 and .6 from left to right, respectively, all cases of moderate dependency.

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Insert Table 2 About Here

.....

.....

Insert Figures 5, 6 and 7 Here

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For example in Figure 5, the easy item is in the second position in Sequence C and E. In C the item follows the medium difficulty item while in E, it follows the most difficult item. The ICC of this particular item shifts to become more difficult for all examinees in both sequences, but the shift is more dramatic after following the most difficult item. In the bottom portion

of this figure, the same easy item is now located in the third position of the three-item sequence. In Sequence D, the series begins with the medium difficulty item, followed by the most difficult item. In Sequence F, the order f the first two items is reversed, with the difficult item being first, followed by the medium difficult item in second position. It can be demonstrated that the effect of prior items depends upon the values of $\alpha$ and $\beta$. It can also be seen in the figures that when $\alpha = \beta = .5$, the ICC of the item in the third position will be the same regardless of the sequence (e.g., A and C). Similar results hold for Figures 6 and 7.

## Conclusions and Directions for Future Work

These preliminary results suggest that the degree of shift in an item's ICC can be fairly dramatic when the response to an item is contingent not only on the degree of dependency with previous items but also on the order and characteristics of those items. Although more work needs to be done to verify that the proposed dependency model accurately describes "real" test data, this issue raises the concern that users of IRT models should not accept the concept of local independence on faith for all situations.

The example presented in Figure 1 illustrates how the acceptance of the assumption of local independence might lead to erroneous item calibrations. If item #3 of this example were to appear on some test without the preceding two items in the example, the item parameters and, hence the ICC, could be estimated at values quite different from those that might arise when all three items were presented together. It would be highly probable that an examinee would be influenced by the presentation of and subsequent responses to items #1 and #2, thus influencing his or her response to item #3. Therefore, if the

item had been calibrated within the three-item sequence but later presented, say as a single item in a computer-adaptive testing situation, the item characteristics could be inaccurate and misleading.

Future work involving this dependent IRT model will be continued in several directions. First, we are interested in the identification and estimation of item dependency in simulated and ultimately, real data sets to determine if this model is a valid representation of the violation of local independence. We are also interested in investigating the robustness of several popular nonlinear IRT model estimation computer programs (e.g., LOGIST, BILOG) with the violation of this assumption. And finally the issue of multidimensionality and its relationship to item dependency needs to be investigated. We see all of these issues as important outgrowths of the model, so long as the validity of the model can be established on actual "dependent" item responses.

References

Lazarsfeld, P. F., & Henry, N. W. (1968). Latent structure analysis. Boston: Houghton Mifflin.

Ross, S. (1976). A first course in probability. New York: Macmillan.

Table 1

Item Parameter Values, Logistic Model

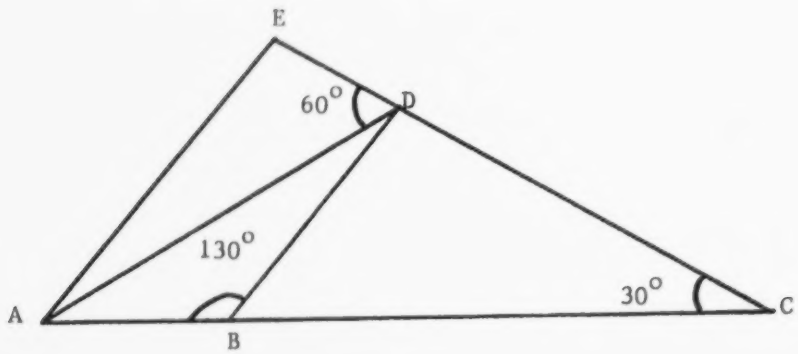
| Item Label | | 1-PLM | 2-PLM | 3-PLM |
|---|---|---|---|---|
| | b | -2.30 | -2.30 | -2.30 |
| EASY | a | 1.00 | .50 | .50 |
| | c | .00 | .00 | .20 |
| | b | .00 | .00 | .00 |
| MEDIUM | a | 1.00 | .75 | .75 |
| | c | .00 | .00 | .20 |
| | b | 2.00 | 2.00 | 2.00 |
| DIFFICULT | a | 1.00 | 1.50 | 1.50 |
| | c | .00 | .00 | .20 |

Table 2

Three-Item Sequences According To Item Difficulty

| Sequence | Item Order by Difficulty | | |
|---|---|---|---|
| A | Easy | -Medium | -Difficult |
| B | Easy | -Difficult | -Medium |
| C | Medium | -Easy | -Difficult |
| D | Medium | -Difficult | -Easy |
| E | Difficult | -Easy | -Medium |
| F | Difficult | -Medium | -Easy |

F I G U R E 1




Given: $\overline{AE}$ is parallel to $\overline{BD}$

< ABD = 130°

< EDA = 60°

< BCD = 30°

1. What is < CBD?

   A. 50°
   B. 60°
   C. 65°
   D. 100°

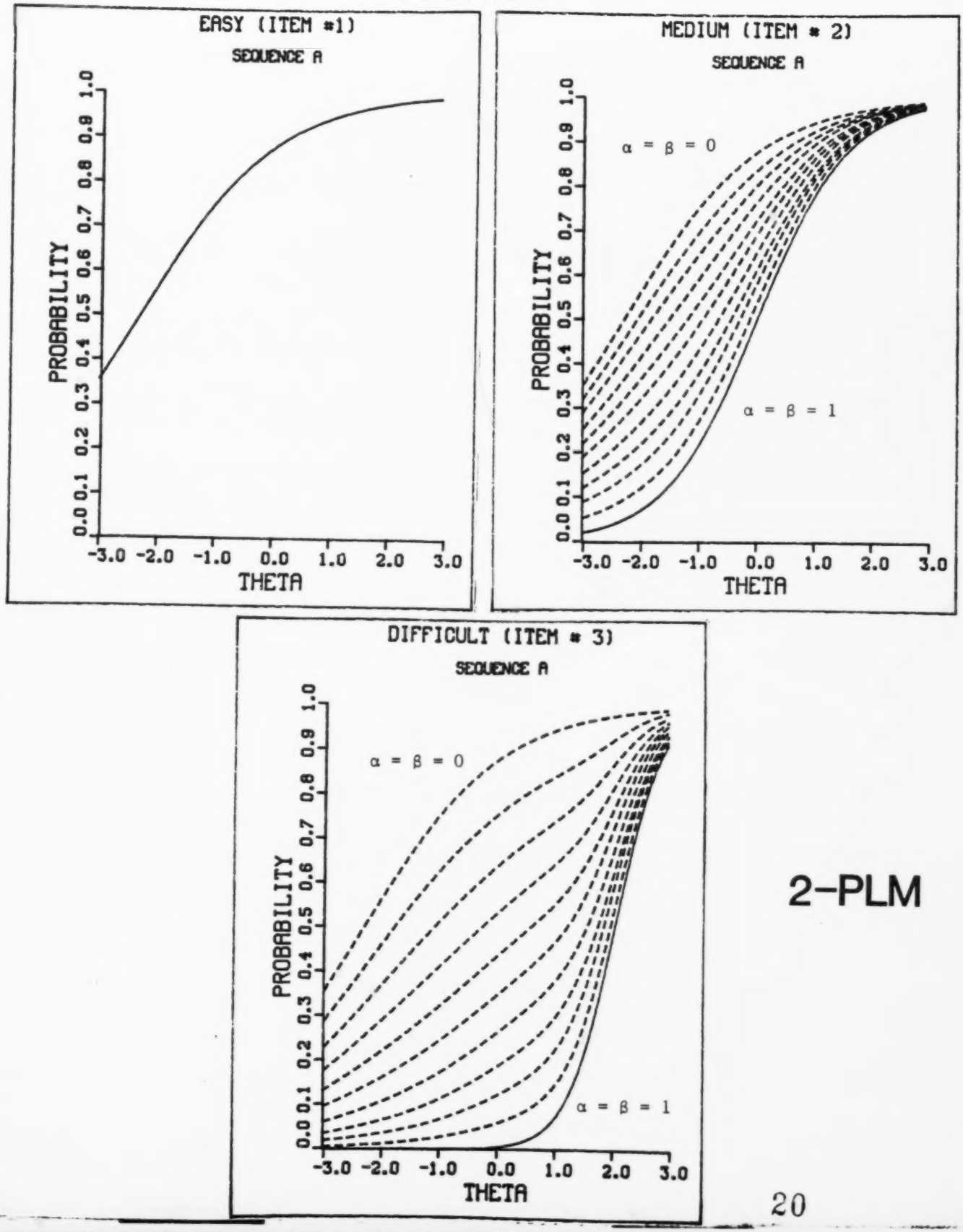
2. Find < BDC.

   A. 130°
   B. 100°
   C. 90°
   D. 85°

3. Find < EAD.

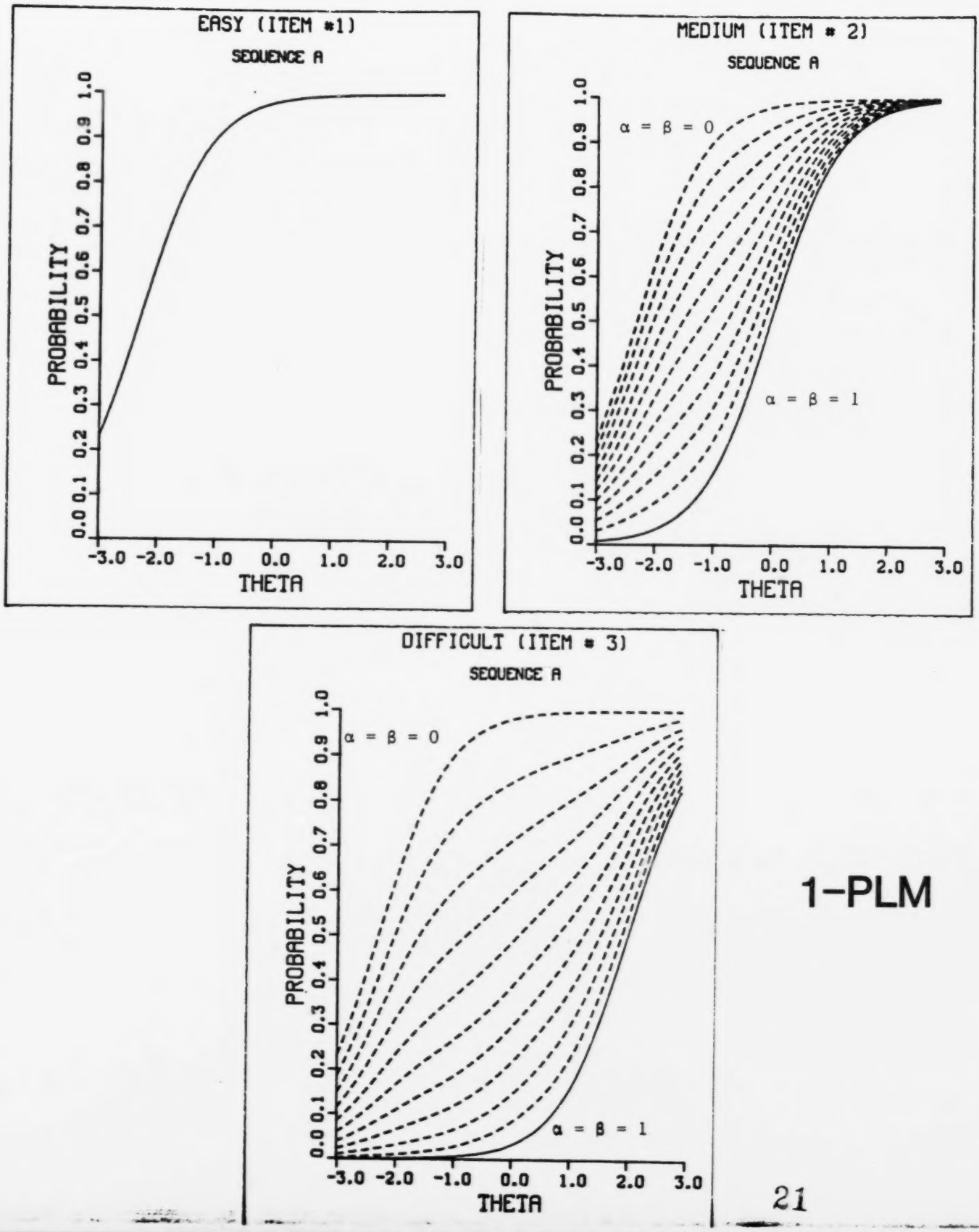
   A. 20°
   B. 30°
   C. 35°
   D. 60°

FIGURE 2




2-PLM

F I G U R E 3




1-PLM

F I G U R E 4

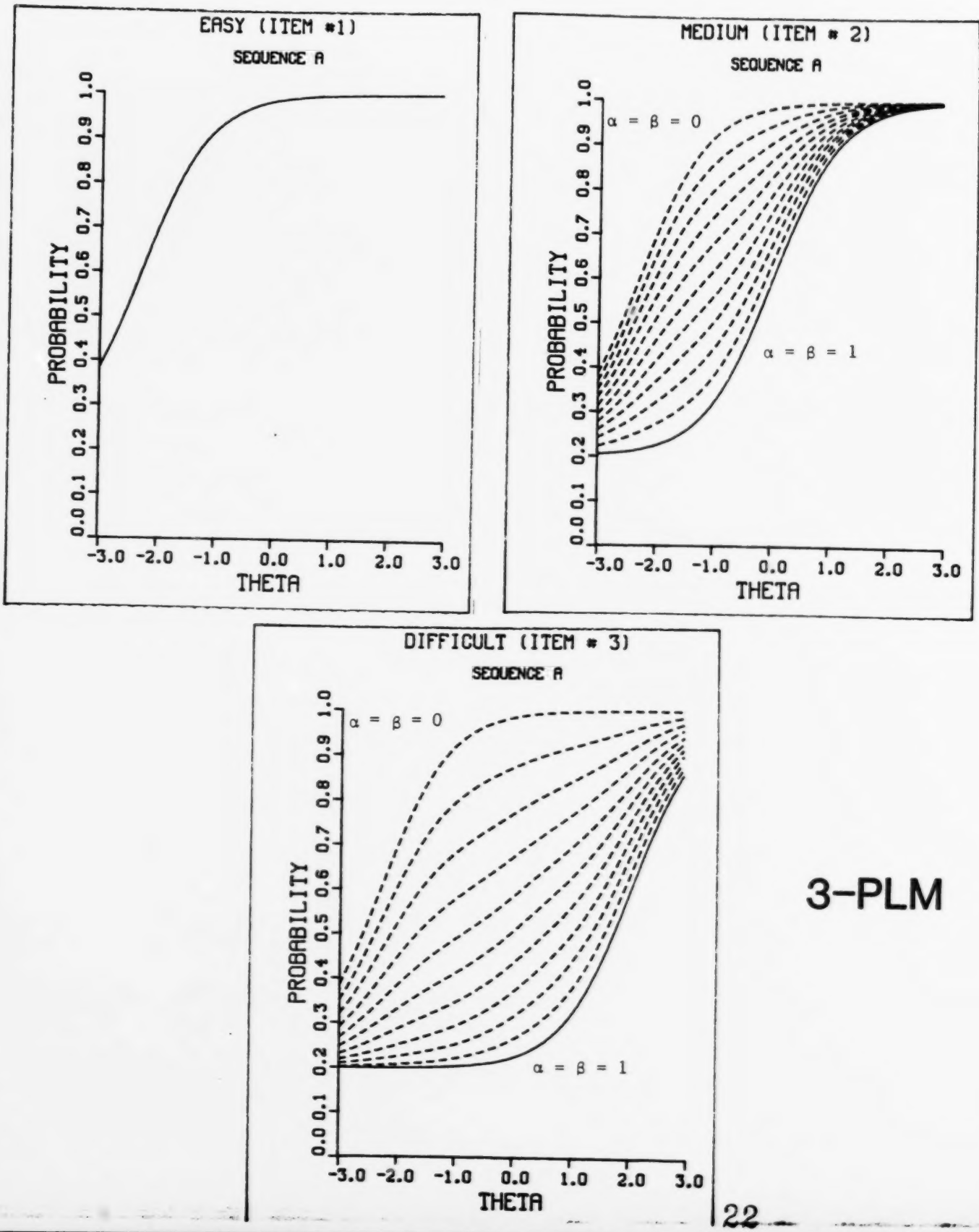
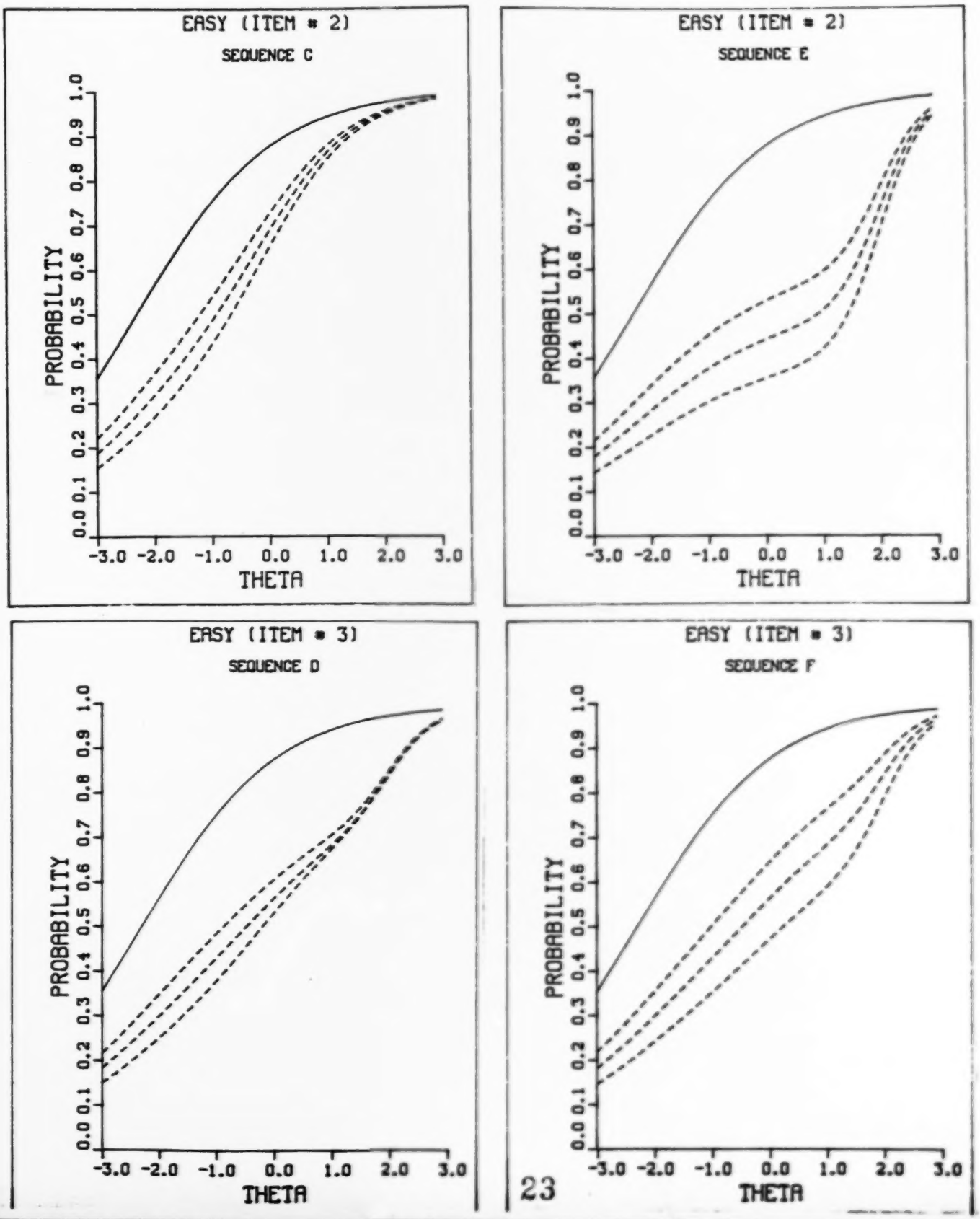
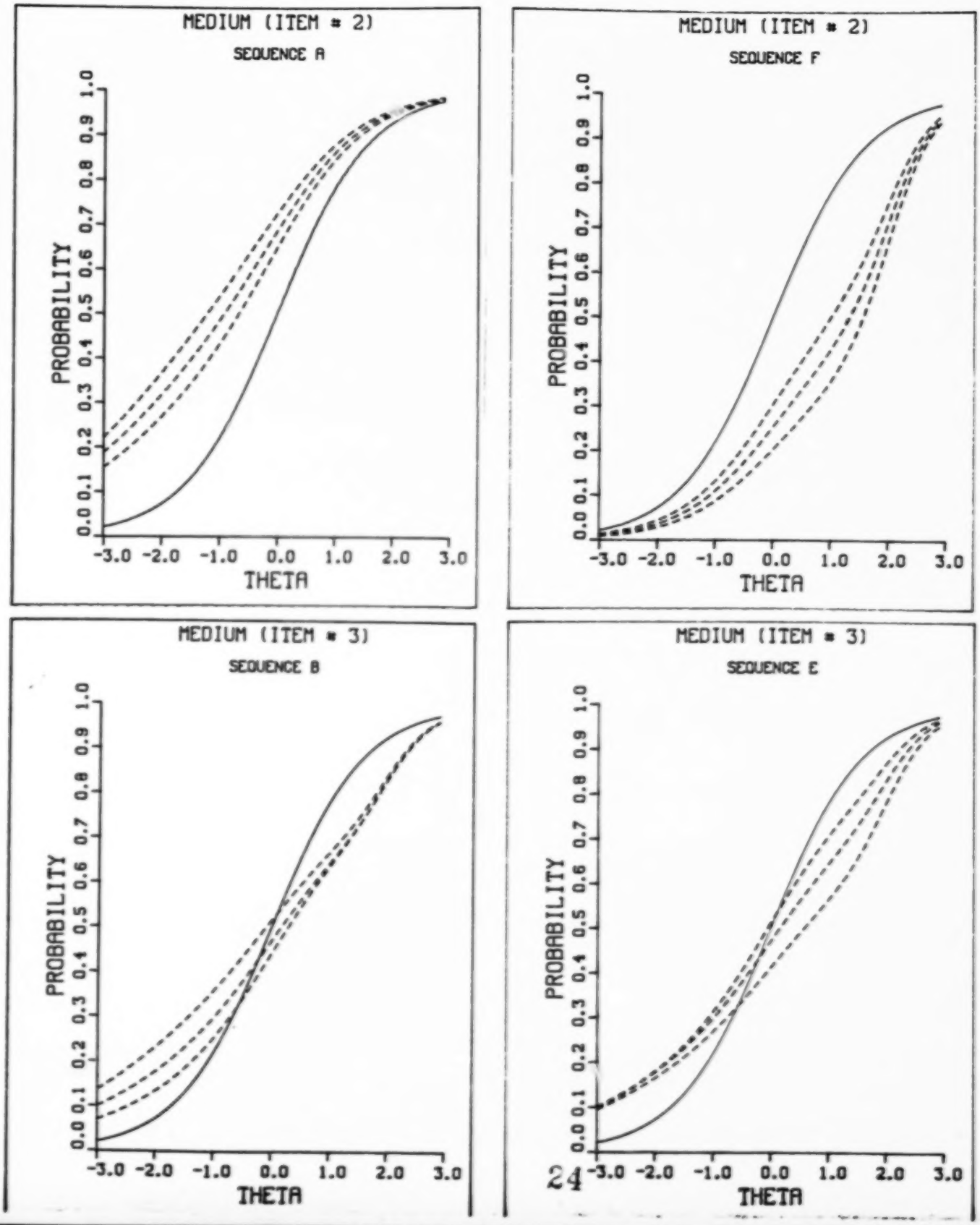

EASY (ITEM #1)
SEQUENCE A
PROBABILITY
0.0 0.1 0.2 0.3 0.4 0.5 0.6 0.7 0.8 0.9 1.0
-3.0 -2.0 -1.0 0.0 1.0 2.0 3.0
THETA
MEDIUM (ITEM # 2)
SEQUENCE A
α = β = 0
α = β = 1
PROBABILITY
0.0 0.1 0.2 0.3 0.4 0.5 0.6 0.7 0.8 0.9 1.0
-3.0 -2.0 -1.0 0.0 1.0 2.0 3.0
THETA
DIFFICULT (ITEM # 3)
SEQUENCE A
α = β = 0
α = β = 1
PROBABILITY
0.0 0.1 0.2 0.3 0.4 0.5 0.6 0.7 0.8 0.9 1.0
-3.0 -2.0 -1.0 0.0 1.0 2.0 3.0
THETA
3-PLM

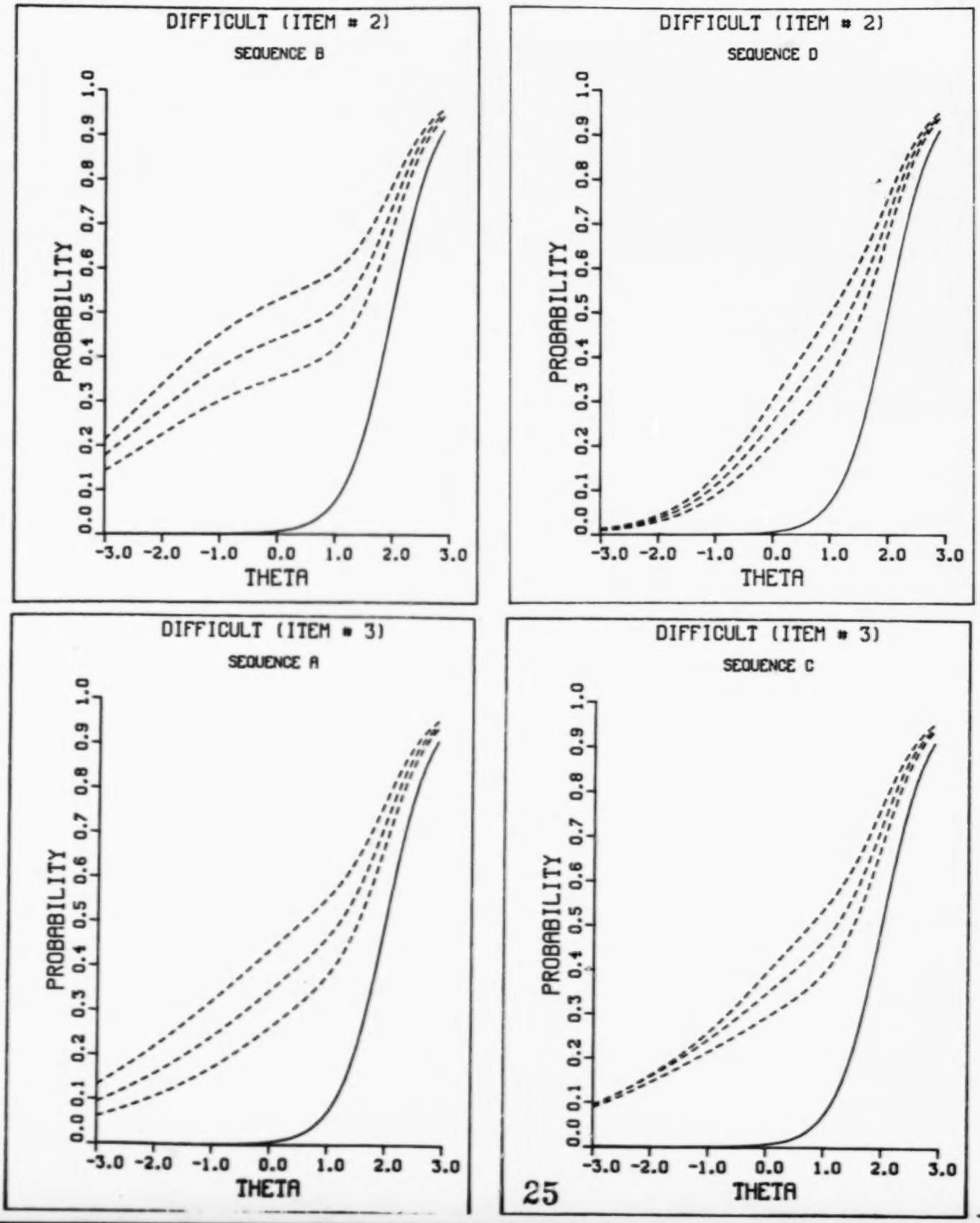
FIGURE 5


EASY (ITEM # 2)
SEQUENCE C
PROBABILITY
0.0 0.1 0.2 0.3 0.4 0.5 0.6 0.7 0.8 0.9 1.0
-3.0 -2.0 -1.0 0.0 1.0 2.0 3.0
THETA
EASY (ITEM # 2)
SEQUENCE E
PROBABILITY
0.0 0.1 0.2 0.3 0.4 0.5 0.6 0.7 0.8 0.9 1.0
-3.0 -2.0 -1.0 0.0 1.0 2.0 3.0
THETA
EASY (ITEM # 3)
SEQUENCE D
PROBABILITY
0.0 0.1 0.2 0.3 0.4 0.5 0.6 0.7 0.8 0.9 1.0
-3.0 -2.0 -1.0 0.0 1.0 2.0 3.0
THETA
EASY (ITEM # 3)
SEQUENCE F
PROBABILITY
0.0 0.1 0.2 0.3 0.4 0.5 0.6 0.7 0.8 0.9 1.0
-3.0 -2.0 -1.0 0.0 1.0 2.0 3.0
THETA

F I G U R E 6


MEDIUM (ITEM # 2)
SEQUENCE A
PROBABILITY
0.0 0.1 0.2 0.3 0.4 0.5 0.6 0.7 0.8 0.9 1.0
-3.0 -2.0 -1.0 0.0 1.0 2.0 3.0
THETA
MEDIUM (ITEM # 2)
SEQUENCE F
PROBABILITY
0.0 0.1 0.2 0.3 0.4 0.5 0.6 0.7 0.8 0.9 1.0
-3.0 -2.0 -1.0 0.0 1.0 2.0 3.0
THETA
MEDIUM (ITEM # 3)
SEQUENCE B
PROBABILITY
0.0 0.1 0.2 0.3 0.4 0.5 0.6 0.7 0.8 0.9 1.0
-3.0 -2.0 -1.0 0.0 1.0 2.0 3.0
THETA
MEDIUM (ITEM # 3)
SEQUENCE E
PROBABILITY
0.0 0.1 0.2 0.3 0.4 0.5 0.6 0.7 0.8 0.9 1.0
-3.0 -2.0 -1.0 0.0 1.0 2.0 3.0
THETA

F I G U R E 7


DIFFICULT (ITEM # 2)
SEQUENCE B
PROBABILITY
0.0 0.1 0.2 0.3 0.4 0.5 0.6 0.7 0.8 0.9 1.0
-3.0 -2.0 -1.0 0.0 1.0 2.0 3.0
THETA
DIFFICULT (ITEM # 2)
SEQUENCE D
PROBABILITY
0.0 0.1 0.2 0.3 0.4 0.5 0.6 0.7 0.8 0.9 1.0
-3.0 -2.0 -1.0 0.0 1.0 2.0 3.0
THETA
DIFFICULT (ITEM # 3)
SEQUENCE A
PROBABILITY
0.0 0.1 0.2 0.3 0.4 0.5 0.6 0.7 0.8 0.9 1.0
-3.0 -2.0 -1.0 0.0 1.0 2.0 3.0
THETA
DIFFICULT (ITEM # 3)
SEQUENCE C
PROBABILITY
0.0 0.1 0.2 0.3 0.4 0.5 0.6 0.7 0.8 0.9 1.0
-3.0 -2.0 -1.0 0.0 1.0 2.0 3.0
THETA